

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 1	Daily Tutorial Sheet 5
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61.(B) $3x^2 + 2x + p(p-1) = 0$

For roots to be of opposite sign : Product of Roots < 0

$$\Rightarrow \frac{p(p-1)}{3} < 0 \Rightarrow \begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ | \quad | \\ 0 \quad 1 \end{array}$$

$$\Rightarrow p \in (0, 1)$$

62.(C) $x^2 + 2(k+1)x + 9k - 5 = 0$

Sum of roots < 0 ; Products of roots > 0 $D \geq 0$

$$\Rightarrow -2(k+1) < 0$$

$$\Rightarrow 9k - 5 > 0$$

$$4(k+1)^2 - 4(9k-5) \geq 0$$

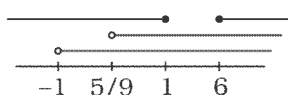
$$\Rightarrow k > -1$$

$$\Rightarrow k > 5/9$$

$$\Rightarrow k^2 - 7k + 6 \geq 0$$

$$(k-1)(k-6) \geq 0$$

Combined Solution :



$$k \in \left(\frac{5}{9}, 1\right] \cup [6, \infty)$$

63.(D) Let $f(x) = x^2 + ax + b$. Clearly $f(0) = b \Rightarrow b$ is an integer.

Also $f(1) = 1 + a + b \Rightarrow a$ is an integer

Hence both a and b are integers

64.(A) $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} - 4 \leq 0 \Rightarrow \frac{2x^2 + 3x - 27}{x^2 - 2x + 6} \leq 0$

Denominator $x^2 - 2x + 6 > 0 \forall x \in \mathbb{R}$ ($\because a > 0$ and $D < 0$)

$$\text{Then, } 2x^2 + 3x - 27 \leq 0 \Rightarrow (2x+9)(x-3) \leq 0 \Rightarrow -\frac{9}{2} \leq x \leq 3 \Rightarrow 0 \leq x^2 \leq \frac{81}{4}$$

$$\therefore \left(4x^2\right)_{\max} = 4\left(-\frac{9}{2}\right)^2 = 81 \text{ and } \left(4x^2\right)_{\min} = 4(0) = 0$$

65.(C) Given inequation is $\frac{1}{x^2 - x + 1} \geq 4$

$$\Rightarrow x^2 - x + 1 \leq \frac{1}{4} \quad \left(\because x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4} \forall x \in \mathbb{R} \right) \Rightarrow x^2 - x + \frac{1}{4} + \frac{3}{4} \leq \frac{1}{4}$$

$$\Rightarrow \left(x - \frac{1}{2}\right)^2 \leq \frac{1}{4} - \frac{3}{4} \Rightarrow \left(x - \frac{1}{2}\right)^2 \leq -\frac{1}{2} \text{ which is not true for any real } x.$$

66.(A) $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2 = \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)^2 - 2 = \left[\frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)}\right]^2 - 2 \dots(i)$

Now, $\alpha + \beta = p$ and $\alpha\beta = q \Rightarrow \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \left[\frac{p^2 - 2q}{q} \right]^2 - 2 = \frac{p^4}{q^2} + 4 - \frac{4p^2}{q} - 2 = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$ [Using (i)]

67.(A) For y to be real $\frac{(x+1)(x-3)}{x-2} \geq 0$

$$\Rightarrow \begin{array}{c} - \quad + \quad - \quad + \\ | \quad | \quad | \quad | \\ -1 \quad 2 \quad 3 \end{array}$$

Now $f(x) \geq 0 \forall x \in [-1, 2) \cup [3, \infty)$

68.(B) $\alpha + \beta = -\frac{b}{a}$

$$\alpha + \beta + 2h = -\frac{q}{p} \Rightarrow 2h = \frac{b}{a} - \frac{a}{p}$$

$$\therefore h = \frac{1}{2} \left(\frac{b}{a} - \frac{a}{p} \right)$$

69.(C) $\alpha + \beta = -b$ and $\alpha\beta = 1$

Now, for the required equation sum of roots $= -\left(\alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} \right) = -\left(\alpha + \beta + \frac{\alpha + \beta}{\alpha\beta} \right) = -\left[-b + \frac{-b}{1} \right] = 2b$

Product of roots $= -\left(\alpha + \frac{1}{\beta} \right) \times \left(-\left[\beta + \frac{1}{\alpha} \right] \right) = \alpha\beta + \frac{1}{\alpha\beta} + 2 = 1 + \frac{1}{1} + 2 = 4$

So, the required equation is $x^2 - 2bx + 4 = 0$

Alternate solution:

$$\alpha\beta = 1 \Rightarrow \frac{1}{\beta} = \alpha \text{ and } \frac{1}{\alpha} = \beta$$

Hence, new equation has roots -2α and -2β and can be obtained by substituting $y = -2x$ i.e.

$$x = -\frac{y}{2} \text{ in given equation } \frac{y^2}{4} - \frac{by}{2} + 1 = 0$$

$$\Rightarrow y^2 - 2by + 4 = 0.$$

70.(B) Given, $3^x + \frac{3}{3^x} - 4 < 0 \Rightarrow 3^{2x} + 3 - 4 \cdot 3^x < 0 \Rightarrow (3^x - 1)(3^x - 3) < 0$

$$1 < 3^x < 3 \Rightarrow 0 < x < 1 \quad \therefore \text{The solution set is } (0, 1)$$

71.(C) Putting $y = (7 + 4\sqrt{3})^{|x|-8}$, we get from the given equation

$$y + \frac{1}{y} = 14 \Rightarrow y^2 - 14y + 1 = 0 \Rightarrow y = \frac{14 \pm \sqrt{196 - 4}}{2} = \frac{14 \pm 8\sqrt{3}}{2} = 7 \pm 4\sqrt{3} = (7 + 4\sqrt{3})^{\pm 1}$$

Thus, $(7 + 4\sqrt{3})^{|x|-8} = (7 + 4\sqrt{3})^{\pm 1} \Rightarrow |x| - 8 = \pm 1 \Rightarrow |x| = 9, 7 \Rightarrow x = \pm 9, \pm 7$

72.(B) Clearly $|x-2| = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = -2 \text{ or } 1 \Rightarrow |x-2| = 1$ [$\because |x-2|$ cannot be -ve]

$$\therefore x-2 = \pm 1 \Rightarrow x = 2 \pm 1 = 3 \text{ or } 1. \text{ Their sum} = 3 + 1 = 4$$

73.(C) Let $f(x) = (x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a)$

$$f(a) = (a-b)(a-c), f(b) = (b-c)(b-a), f(c) = (c-a)(c-b)$$

If $a = b$ (WLOG)

Then $x = a$ is a root for sure and thus the given equation will always have real roots.

Now, we will handle the case when a, b, c are distinct.

Since $f(x)$ is symmetric with respect to a, b and c , let us assume that $a < b < c$. Then $f(a) > 0, f(b) < 0$ and $f(c) > 0$.

$$\Rightarrow f(a) \cdot f(b) < 0 \quad \text{and} \quad f(b) \cdot f(c) < 0$$

$$\Rightarrow f(x) \text{ has one root in } x \in (a, b) \text{ and other root in } x \in (b, c) \Rightarrow f(x) \text{ has real roots.}$$

74.(B) $\frac{a}{x-a} + \frac{b}{x-b} = 1$ gives $a(x-b) + b(x-a) = (x-a)(x-b)$ or $x^2 - 2(a+b)x + 3ab = 0$

Since roots are equal in magnitude but opposite in sign, sum of roots $= 0 = 2(a+b) \Rightarrow a+b=0$

75.(AC) Given $(x-1)(x-2) + (x-1)(x-3) + (x-2)(x-3) = 0$

$$\therefore 3x^2 - 12x + 11 = 0$$

$$D > 0$$

Roots are real and unequal.